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DAMPING AND SCATTERING OF ELECTROMAGNETIC WAVES BY SMALL FERRITE SPHERES SUSPENDED IN AN INSULATOR

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SUMMARY

The intentional degradation of electromagnetic waves by their penetration into a media comprised of somewhat sparsely distributed energy absorbing ferrite spheres suspended in an electrical insulator is investigated. Results are presented in terms of generalized parameters involving wave length and sphere size, sphere resistivity, permeability, and spacing; and their influence on dissipation of wave power by eddy currents, magnetic hysteresis, and scattering is shown.

SYMBOLS

\mathcal{A}	$a^2 \omega \mu / \tau$
\mathcal{A}_0	$a^2 \omega \mu_0 / \tau$
a	radius of sphere
a_ℓ	coefficients in equation (8)
\vec{B}	magnetic flux density
b_ℓ	coefficients in equation (7)
C_a	$\cosh (2a \operatorname{Re} \sqrt{j p})$
c_0	speed of wave travel
c_a	$\cos (2a \operatorname{Im} \sqrt{j p})$
\vec{D}	electric displacement current
\mathcal{D}	constant defined in equation (6)
\vec{E}	electric field vector
e	2.71828
\vec{H}	magnetic inductance
$H^{(1)}$	Hankel function of first kind
Im	imaginary part of a complex quantity
J	Bessel function of first kind
K	power loss constants in equation (2)
k	complex wave number
N	number density of spheres per unit volume
P	power
p	parameter $\omega \mu / \tau$
Re	real part of a complex quantity

S_a	$\sinh (2a\mathbf{Re} \sqrt{j\mathbf{p}})$
Shi	hyperbolic sine integral
Si	sine integral
s	average spacing between center of spheres
s_a	$\sin (2a\mathbf{Im} \sqrt{j\mathbf{p}})$
t	time
U_n	$\mu_n - 1$
U_{gn}	$ \mu_n ^2 + (1 - 2 \mathbf{Re} \mu_n)$
x	distance in direction of wave travel
α	$2 \frac{\mathbf{Re} \sqrt{j\mathbf{p}}}{\sqrt{ \mathbf{p} }}$
β	$2 \frac{\mathbf{Im} \sqrt{j\mathbf{p}}}{\sqrt{ \mathbf{p} }}$
λ	wavelength
μ	permeability inside spheres, $\mu' - j\mu''$
μ_n	normalized permeability, μ/μ_o
μ_o	permeability of material in which spheres are immersed
τ	resistivity
ω	angular velocity

Subscripts

A	applied wave
ec	eddy current
hy	hysteresis

Superscripts

'	real part of complex quantity
"	imaginary part of complex quantity

INTRODUCTION

Ferromagnetic oxides with their low electrical conductivity and large skin depth can be effective in absorbing electromagnetic wave energy (refs. 1 and 2). Their low weight densities, approximately 60 percent that of iron (ref. 3), make them attractive for flight applications. Spheres will be representative of the absorbent particles in the model compositions herein. Eddy currents are induced in the spheres by the applied electromagnetic waves resulting in dissipation of field energy by electrical resistance. Magnetic hysteresis phenomena cause additional losses. This evolves from the continual distortion of the crystalline structure of the Weiss domains and spin reorientation of the bound electrons induced by the waves (ref. 2, p. 16).

The intensity of interaction and resulting scattering of a plane electromagnetic wave by a conducting sphere falls off rapidly as the cube of the outside distance from the center of the sphere (ref. 4). This allows approximation of the multiple sphere problem by a succession of binary interactions except in the case of closely packed spheres.

Absorption of electromagnetic wave energy by small individual ferrite spheres was treated in reference 5. Two algebraic equations were derived expressing dimensionless power dissipation parameters for eddy currents and for hysteresis loss phenomena. These equations include complex magnetic permeability, μ , normalized by the permeability of the surrounding media, μ_o , and a parameter of variables, $a^2\omega\mu_o/\tau$ involving angular wave frequency, ω , sphere radius, a , sphere resistivity, τ , as well as μ_o . The present effort also includes power loss by inelastic scattering of the incident waves by the ferrite spheres and is treated by use of a complex wave number, k , (ref. 6).

Particles used in composites to isolate electronic equipment, as well as in wall coatings to damp unwanted electromagnetic radiation, are usually much smaller than the radiation wave length. At such low ratios of particle size to wave length diffraction plays a minor role and has been omitted.

ANALYSIS

The essential power balance equation for absorption of electromagnetic energy by means of eddy currents, P_{ec} , and magnetic hysteresis P_{hy} , induced in ferrite spheres can be expressed as

$$\frac{1}{2} \left(\vec{E}_A \cdot \frac{\partial \vec{D}_A}{\partial t} + \vec{H}_A \cdot \frac{\partial \vec{B}_A}{\partial t} \right) = \vec{H}_A \cdot \frac{\partial \vec{B}_A}{\partial t} = \frac{1}{2\mu_A} \frac{\partial B_A^2}{\partial t} = -N(P_{ec} + P_{hy}) \quad (3)$$

The applied electric and magnetic fields contribute equally in the initial power source. Here \vec{H}_A and \vec{B}_A are the magnetic induction and flux density of the wave whereas \vec{E}_A and \vec{D}_A are the electric field intensity and electric displacement vectors respectively. N is the number density of spheres per unit volume of composite. Power densities P_{ec} and P_{hy} are equal to constants K_{ec} and K_{hy} respectively times the square of the magnetic field of the applied wave, B_A , as in equations (33) and (35) of reference 5.

Including an additional power loss term, $P_s = B_A^2 K_s$ to represent elastic scattering of the incident wave, and therefore further loss of return signal, gives

$$\frac{dB_A^2}{B_A^2} = -\frac{2\mu_A N}{c_o} (K_{ec} + K_{hy} + K_s) dx \quad (2)$$

where $c_o = dx/dt$ is approximated by the speed of wave travel in the carrier material in which the spheres are immersed.

Integrating both sides of (2) gives the stopping power formula

$$B_A(x) = B_A(0)e^{-\mu_A N(K_{ec} + K_{hy} + K_s)x/c} \quad (3)$$

By (ref. 5) the constant, K_{ec} , for eddy currents, is

$$K_{ec} = \frac{P_{ec}}{B_A^2} = \frac{3\pi\omega^2 a^5 |\mu_n|^2 |\mathcal{A}|}{\tau \mathcal{D} \operatorname{Re} \mathcal{A}} \left[S_a \operatorname{Im} \sqrt{j\mathcal{A}} + s_a \operatorname{Re} \sqrt{j\mathcal{A}} - (C_a - c_a) \frac{\operatorname{Re} \mathcal{A}}{|\mathcal{A}|} \right] \quad (4)$$

and for hysteresis

$$K_{hy} = \frac{P_{hy}}{B_A^2} = \frac{9\pi\omega^2 a^5 \mu'' |\mu_n| \sqrt{|\mathcal{A}|}}{\tau \mathcal{D}} \left\{ \left(\frac{\alpha^2 + \beta^2}{6} - 1 \right) \frac{C_a + c_a}{\sqrt{|\mathcal{A}|}} + \frac{1}{3} \left[\frac{\alpha S_a - \beta s_a}{|\mathcal{A}|} - \frac{C_a - c_a}{|\mathcal{A}|^{3/2}} + \frac{S_a}{\alpha} - \frac{s_a}{\beta} \right] \right. \\ \left. + \left(\frac{4}{3} + \frac{\beta^2}{6} - \frac{\alpha^2}{2} \right) \alpha \operatorname{Shi}(2 \operatorname{Re} \sqrt{\mathcal{A}}) - \left(\frac{4}{3} + \frac{\alpha^2}{6} - \frac{\beta^2}{2} \right) \beta \operatorname{Si}(2 \operatorname{Im} \sqrt{\mathcal{A}}) \right\} \quad (5)$$

where parameter $\mathcal{A} = a^2 \omega \mu / \tau$, normalized complex permeability $\mu_n = (\mu' - j\mu'')/\mu_0$, $j = \sqrt{-1}$, and μ_0 is permeability of the media in which the spheres are suspended.

The constant \mathcal{D} in the denominator of equations (4) and (5) is

$$\mathcal{D} = U_{gn}^2 |\mathcal{A}| (C_a + c_a) + \left[U_{gn}^2 + 2 \operatorname{Im}(\mathcal{A} \bar{\mu}_n) + |\mathcal{A}|^2 - 2 \operatorname{Im} \mathcal{A} \right] (C_a - c_a) \\ - \sqrt{2} \left\{ U_{gn}^2 \left[(S_a + s_a) \operatorname{Re} \sqrt{\mathcal{A}} - (S_a - s_a) \operatorname{Im} \sqrt{\mathcal{A}} \right] \right. \\ \left. - (S_a - s_a) \operatorname{Re} (U_n \bar{\mathcal{A}} \sqrt{\mathcal{A}}) - (S_a + s_a) \operatorname{Im} (U_n \bar{\mathcal{A}} \sqrt{\mathcal{A}}) \right\} \quad (6)$$

where $U_n = \mu_n - 1$, $U_{gn}^2 = |\mu_n|^2 + (1 - 2 \operatorname{Re} \mu_n)$, $p = \omega \mu / \tau$, $C_a = \cosh(2a \operatorname{Re} \sqrt{j p})$, $c_a = \cos(2a \operatorname{Im} \sqrt{j p})$, $S_a = \sinh(2a \operatorname{Re} \sqrt{j p})$, and $s_a = \sin(2a \operatorname{Im} \sqrt{j p})$.

Utilizing the scattering cross section, σ , of page 236 of reference 7 or page 449 of reference 8 gives

$$K_s = \frac{P_s}{B_A^2} = \frac{c\sigma}{2\mu_A} = \frac{\pi c}{\mu_A k^2} \sum_{\ell=1}^{\infty} (2\ell + 1) (|a_\ell|^2 + |b_\ell|^2) \quad (7)$$

where a_ℓ and b_ℓ are expressed in Bessel functions of complex argument as

$$a_\ell = -\frac{J_{\ell+1/2}(ka)}{H_{\ell+1/2}^{(1)}(ka)}, \quad b_\ell = -\frac{(\ell - 1)J_{\ell+1/2}(ka) - kaJ_{\ell-1/2}(ka)}{(\ell - 1)H_{\ell+1/2}^{(1)}(ka) - kaH_{\ell-1/2}^{(1)}(ka)}, \quad (8)$$

and

$$k^2 = \left(\frac{\omega}{c} \right)^2 + \frac{j\omega\mu}{r} = \left(\frac{2\pi}{\lambda} \right)^2 + \frac{\omega}{r} (\mu'' + j\mu') .$$

Three coefficients were used in each of the rapidly converging series of equation (7). They can be expressed as

$$a_1 = - \frac{\sin(ka) - ka \cos(ka)}{ka[\sin(ka) - j \cos(ka)]} , \quad (9)$$

$$a_2 = \frac{\frac{3}{ka} \cos(ka) + \left[1 - \frac{3}{(ka)^2} \right] \sin(ka)}{\frac{3}{ka} \cos(ka) + \left[1 - \frac{3}{(ka)^2} \right] \sin(ka) + j \left\{ \frac{3}{ka} \sin(ka) - \left[1 - \frac{3}{(ka)^2} \right] \cos(ka) \right\}} , \quad (10)$$

$$a_3 = \frac{\left[1 - \frac{5!}{8(ka)^2} \right] \cos(ka) + \left[\frac{5!}{8(ka)^3} - \frac{3!}{ka} \right] \sin(ka)}{\left[1 - \frac{5!}{8(ka)^2} + j \left(\frac{3!}{ka} - \frac{5!}{8(ka)^3} \right) \right] [\cos(ka) + j \sin(ka)]} , \quad (11)$$

$$b_1 = \frac{[1 - (ka)^2] \sin(ka) - ka \cos(ka)}{e^{jka} \{ ka - j[(ka)^2 - 1] \}} \quad (12)$$

$$b_2 = \frac{[3(ka)^2 - 6] \sin(ka) - [(ka)^3 - 6ka] \cos(ka)}{e^{jka} \{ (ka)^3 + 6ka + j[6 - (ka)^2] \}} , \quad (13)$$

and

$$b_3 = \frac{(ka)^4 \sin(ka) + 6(ka)^3 \cos(ka) - 21(ka)^2 \sin(ka) - 45[ka \cos(ka) - \sin(ka)]}{e^{jka} \left\{ \frac{9}{2} (ka)^3 + 45ka - j[(ka)^4 - 18(ka)^3 - 3(ka)^2 + 45ka] \right\}} \quad (14)$$

by use of pages 966 and 967 of reference 9.

Next consider the generalized distance to reduce $B_A(x)/B_A(0)$ to $1/e$ and let number density $N = 1/s^3$, where s is the average spacing between centers of spheres. The exponent in equation (3) written in terms of dimensionless parameters, then reduces to -1 when

$$\left(\frac{a}{s}\right)^3 \frac{\omega x}{c} = 1 / \left\{ \frac{\tau(P_{ec} + P_{hy})}{\omega^2 a^5 B^2} (a^2 \omega \mu_A / \tau) + \frac{\lambda/a}{2(ka)^2} \sum_{\ell=1}^3 (2\ell + 1) (|a_\ell|^2 + |b_\ell|^2) \right\} \quad (15)$$

The individual contributions of eddy currents and hysteresis are given in reference 5 over wide ranges of \mathcal{A}_0 .

RESULTS

The generalized distance of wave travel, $(a/s)^3 \omega x/c$, in the composite to reduce $B_A(x)/B_A(0)$ to $1/e$ is plotted versus μ'/μ_0 and μ''/μ_0 for fixed values of $a^2 \omega \mu_0 / \tau$ on three-dimensional plots of figures 1(a) to 2(h). The range of generalized distance decreases with increase of parameter $a^2 \omega \mu_0 / \tau$ for absorption (figs. 1(a) to (d)) as well as for scattering when holding λ/a constant: figures 2(a), (e), and (g) with $\lambda/a = 10^3$ and figures 2(b), (f), and (h) with $\lambda/a = 10^5$.

The peaks of the surfaces move to higher μ'/μ_0 as $a^2 \omega \mu_0 / \tau$ and/or λ/a decrease. Pronounced concave trough-like contours indicate the minimum distance of absorption whereas evident convex patterns give the maximum distances for reduction of scattering. The dashed lines shown are merely diagonals where $\mu' = \mu''$.

As λ/a is decreased from 10^5 the ridges gradually rotate their directions from diagonal to ones more closely parallel to the μ''/μ_0 axis. With further decrease of λ/a the contours flatten out to nearly constant values of generalized distance. This is shown over an especially wide range of λ/a in figures 2(c) to (f). This trend also appears as $a^2 \omega \mu_0 / \tau$ is reduced with λ/a held constant. For example, note the change in curve pattern between figures 2(g), (e), and (a).

The generalized distance to reduce $B_A(x)/B_A(0)$ to $1/e$ by absorption is nonsymmetric about the dashed line due to the lack of symmetry in the contribution from magnetic hysteresis. The results for scattering are symmetric when like values are selected for μ' and μ'' .

CONCLUDING REMARKS

By use of generalized parameters the seven independent variables: sphere radius a , complex permeability $\mu' - j\mu''$, resistivity τ , mean distance between spheres s , electromagnetic field strength B_A , and wave length λ times frequency combine to (1) a generalized distance of wave travel, $(a/s)^3 \omega x/c$, to reduce the applied magnetic field ratio, $B_A(x)/B_A(0)$ to $1/e$ of its initial strength, (2) a power absorption parameter $a^2 \omega \mu_0 / \tau$, and (3) magnetic permeability ratios μ'/μ_0 and μ''/μ_0 .

The range of generalized distance for reducing $B_A(x)/B_A(0)$ to $1/e$ by absorption decreases with increase of $a^2 \omega \mu_0 / \tau$ over its range of 10^{-7} to 10 while the peaks of these plots reduce from the order of 10^7 to 10^3 . The dashed lines shown are merely diagonals where $\mu' = \mu''$.

The three-dimensional plots of generalized distance by scattering, however, are symmetrical about the dashed lines. Also an additional parameter, wave length divided by sphere radius, λ/a , enters as an independent variable for these plots.

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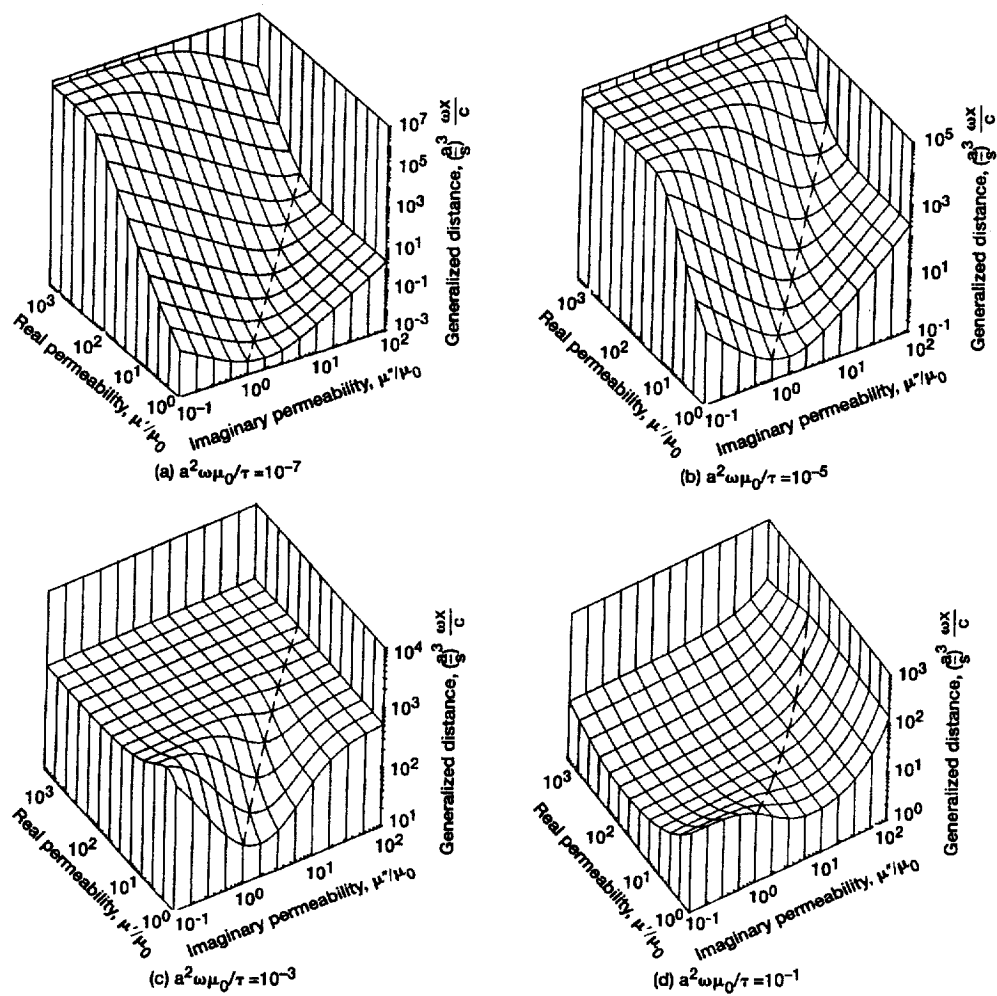


Figure. 1—Generalized distance to reduce $B_A(x)/B_A(0)$ to $1/e$ by absorption.

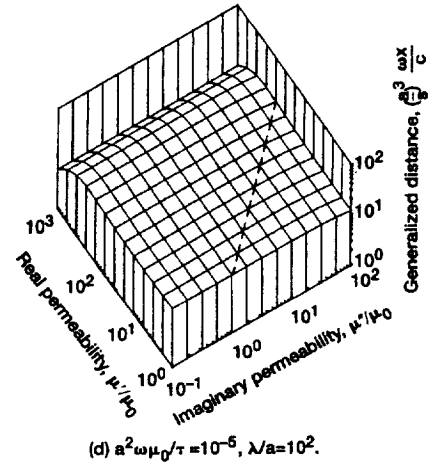
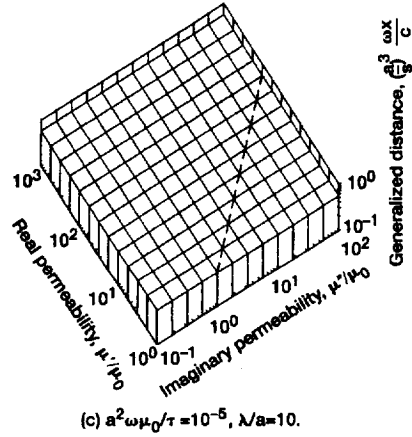
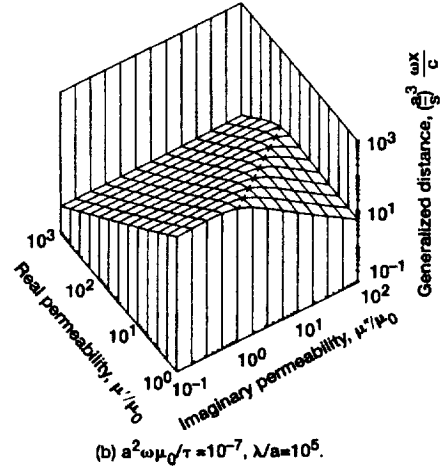
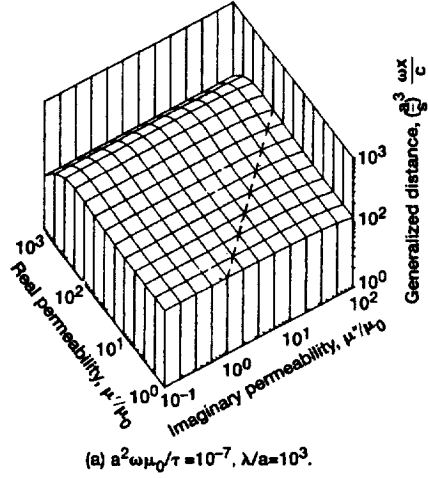


Figure. 2—Generalized distance to reduce $B_A(x)/B_A(0)$ to $1/e$ by scattering.

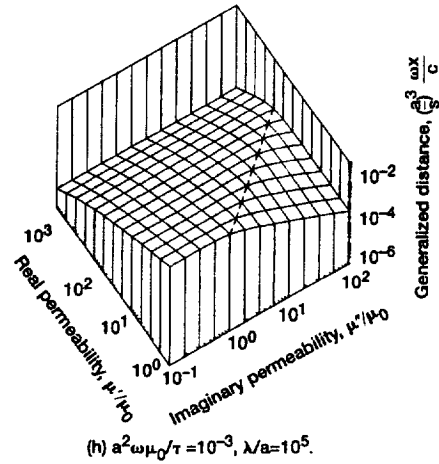
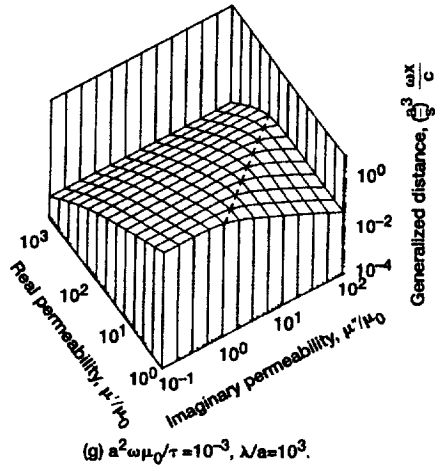
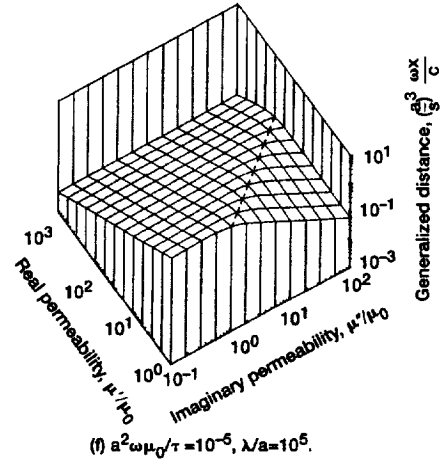
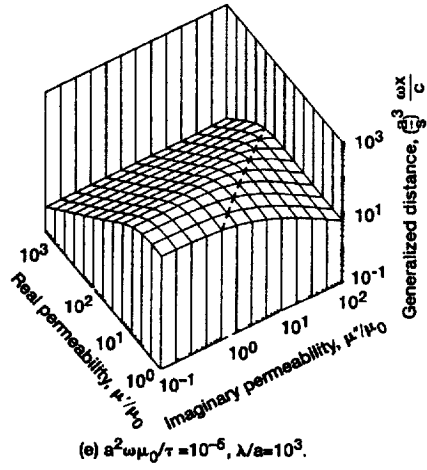


Figure. 2—Concluded